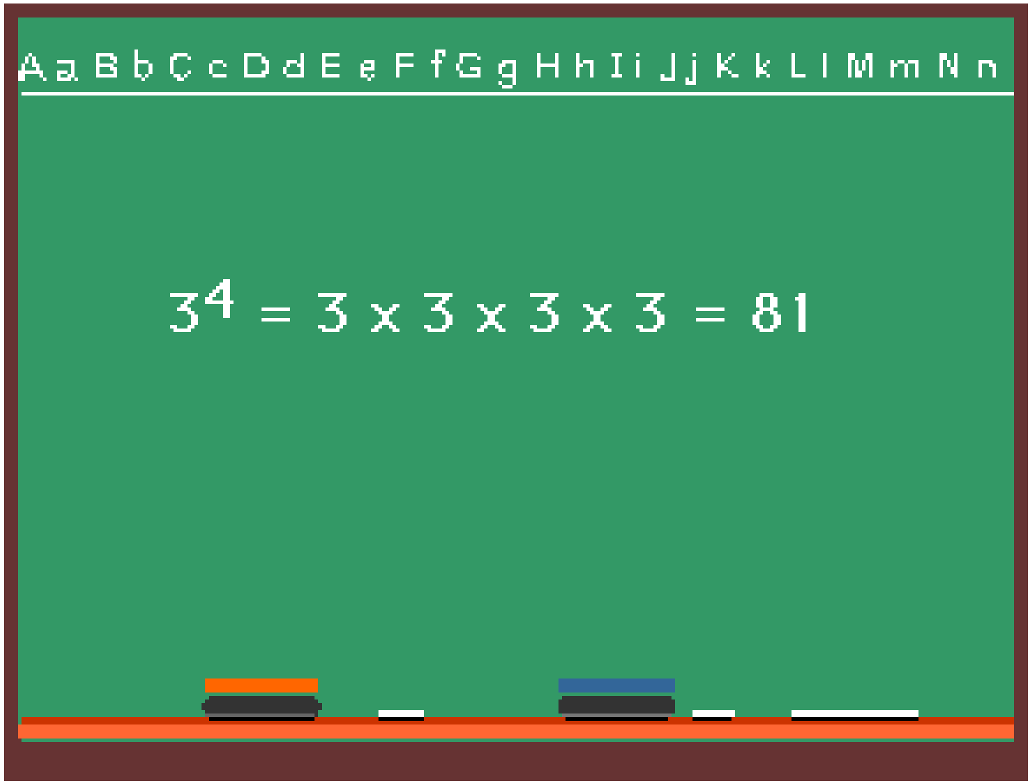
The Laws of Exponents



Exponents

exponent

Power $\rightarrow \left\{ 5^{3} \right.$

base

Example: $125 = 5^3$ means that 5^3 is the exponential form of the number 125.

5^3 means 3 factors of 5 or 5 x 5 x 5

The Laws of Exponents:

#1: Exponential form: ***The exponent of a power indicates how many times the base multiplies itself.***

$$x^n = \underbrace{x \cdot x \cdot x \cdots x \cdot x \cdot x \cdot x}_{n\text{–}times}$$

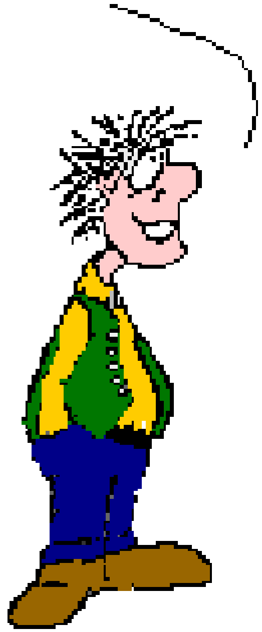
n factors of x

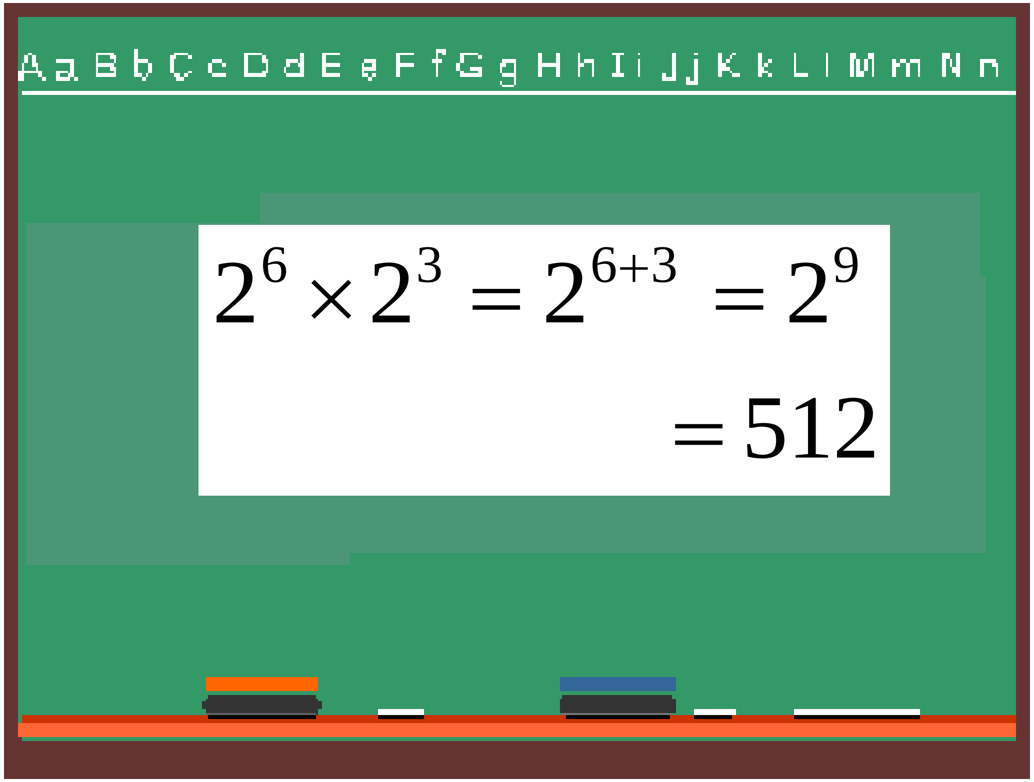
$$\text{Example: } 5^3 = 5 \cdot 5 \cdot 5$$

#2: Multiplying Powers: *If you are multiplying Powers with the same base, KEEP the BASE & ADD the EXPONENTS!*

$$\boxed{x^m \cdot x^n = x^{m+n}}$$

So, I get it! When you multiply Powers, you add the exponents!





$$2^6 \times 2^3 = 2^{6+3} = 2^9$$

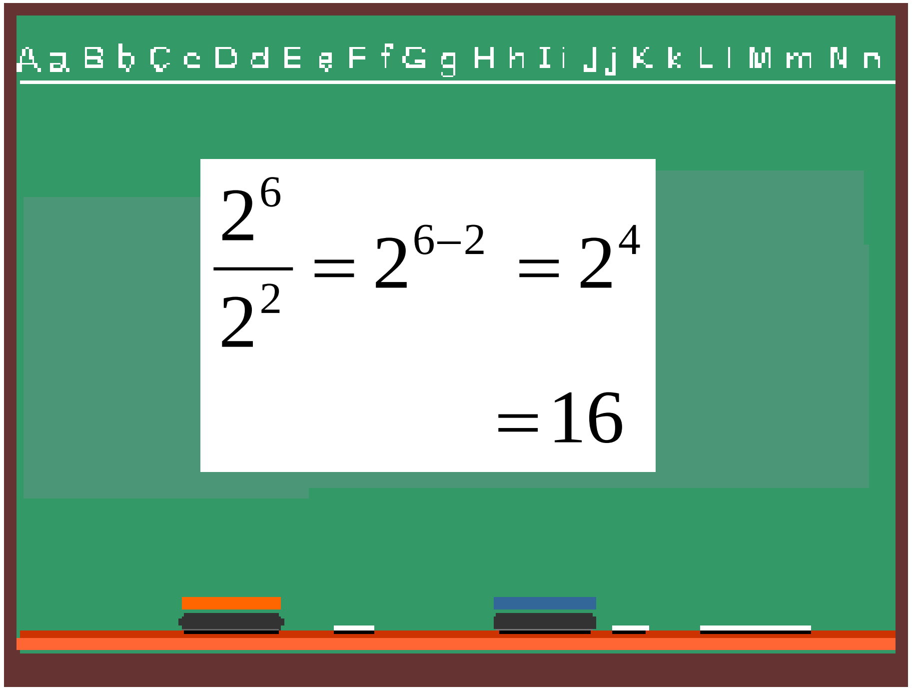
$$= 512$$

#3: Dividing Powers: *When dividing Powers with the same base, KEEP the BASE & SUBTRACT the EXPONENTS!*

$$\frac{x^m}{x^n} = x^m \div x^n = x^{m-n}$$

So, I get it!

When you divide Powers, you subtract the exponents!



Try these:

1. $3^2 \times 3^2 =$

2. $5^2 \times 5^4 =$

3. $a^5 \times a^2 =$

4. $2s^2 \times 4s^7 =$

5. $(-3)^2 \times (-3)^3 =$

6. $s^2 t^4 \times s^7 t^3 =$

7. $\dfrac{s^{12}}{s^4} =$

8. $\dfrac{3^9}{3^5} =$

9. $\dfrac{s^{12} t^8}{s^4 t^4} =$

10. $\dfrac{36a^5 b^8}{4a^4 b^5} =$

SOLUTIONS

1. $3^2 \times 3^2 = 3^{2+2} = 3^4 = 81$

2. $5^2 \times 5^4 = 5^{2+4} = 5^6$

3. $a^5 \times a^2 = a^{5+2} = a^7$

4. $2s^2 \times 4s^7 = 2 \times 4 \times s^{2+7} = 8s^9$

5. $(-3)^2 \times (-3)^3 = (-3)^{2+3} = (-3)^5 = -243$

6. $s^2 t^4 \times s^7 t^3 = s^{2+7} t^{4+3} = s^9 t^7$

SOLUTIONS

7. $$\frac{s^{12}}{s^{4}} = s^{12-4} = s^{8}$$

8. $$\frac{3^{9}}{3^{5}} = 3^{9-5} = 3^{4} = 81$$

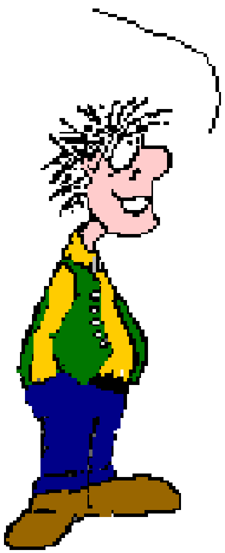
9. $$\frac{s^{12}t^{8}}{s^{4}t^{4}} = s^{12-4}t^{8-4} = s^{8}t^{4}$$

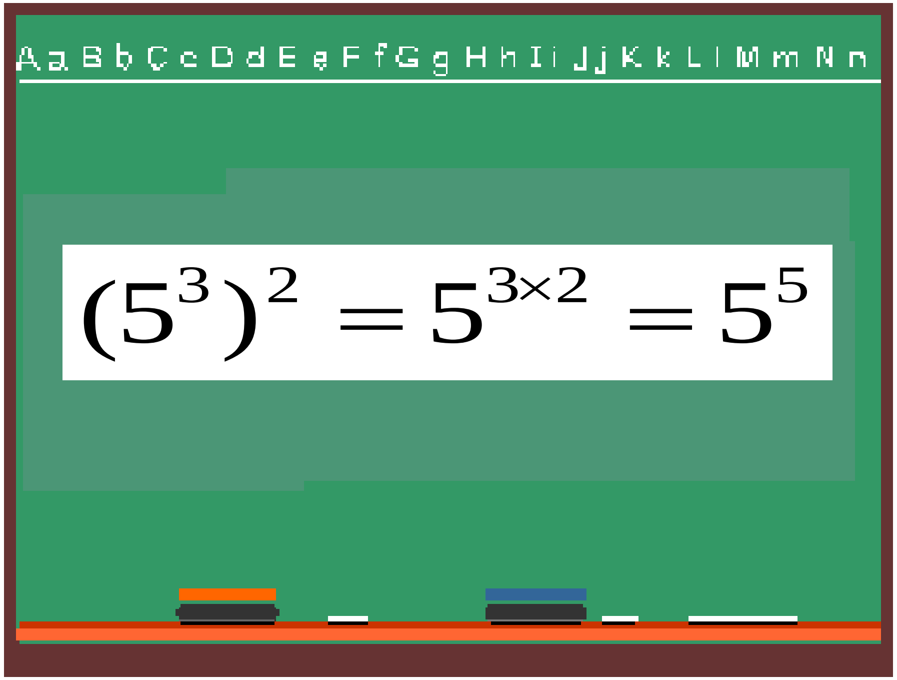
10. $$\frac{36a^{5}b^{8}}{4a^{4}b^{5}} = 36 \div 4 \times a^{5-4}b^{8-5} = 9ab^{3}$$

#4: Power of a Power: *If you are raising a Power to an exponent, you multiply the exponents!*

$$\boxed{\left(x^m\right)^n = x^{mn}}$$

So, when I take a Power to a power, I multiply the exponents

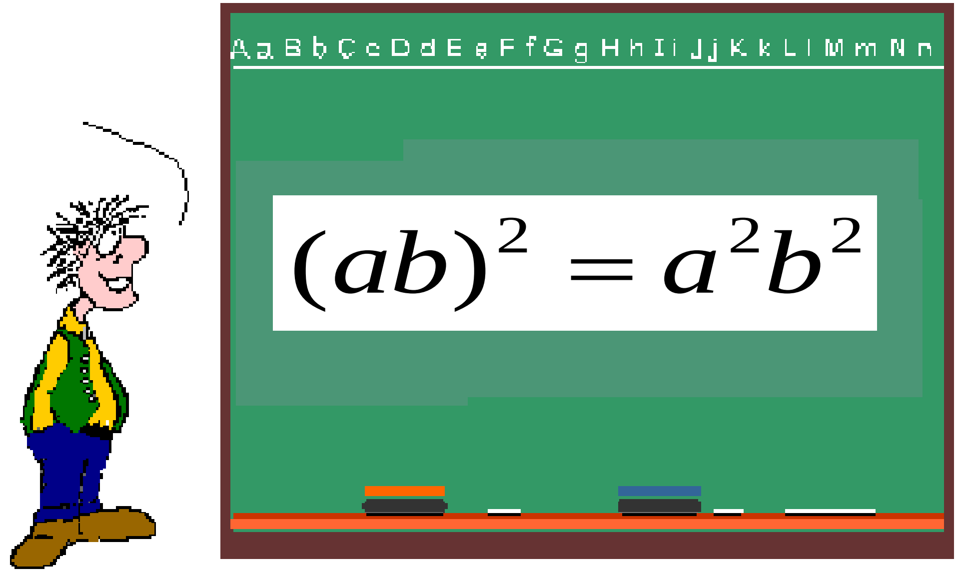




#5: Product Law of Exponents: ***If the product of the bases is powered by the same exponent, then the result is a multiplication of individual factors of the product, each powered by the given exponent.***

$$(xy)^n = x^n \cdot y^n$$

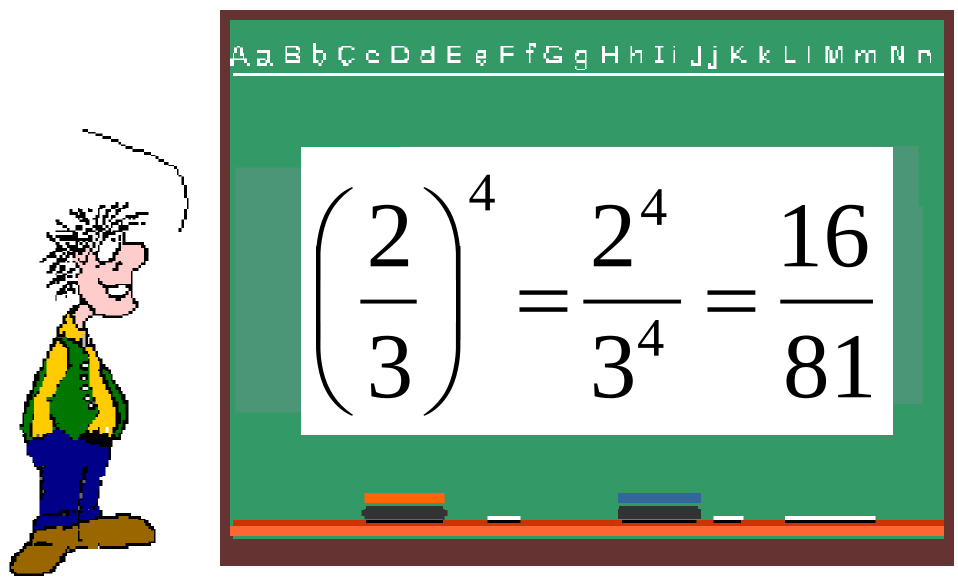
So, when I take a Power of a Product, I apply the exponent to all factors of the product.



#6: Quotient Law of Exponents: *If the quotient of the bases is powered by the same exponent, then the result is both numerator and denominator , each powered by the given exponent.*

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

So, when I take a Power of a Quotient, I apply the exponent to all parts of the quotient.



$$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

Try these:

1. $\left(3^2\right)^5 =$

2. $\left(a^3\right)^4 =$

3. $\left(2a^2\right)^3 =$

4. $\left(2^2 a^5 b^3\right)^2 =$

5. $(-3a^2)^2 =$

6. $\left(s^2 t^4\right)^3 =$

7. $\left(\dfrac{s}{t}\right)^5 =$

8. $\left(\dfrac{3^9}{3^5}\right)^2 =$

9. $\left(\dfrac{st^8}{rt^4}\right)^2 =$

10. $\left(\dfrac{36a^5b^8}{4a^4b^5}\right)^2 =$

SOLUTIONS

1. $\left(3^2\right)^5 = \quad 3^{10}$

2. $\left(a^3\right)^4 = \quad a^{12}$

3. $\left(2a^2\right)^3 = \quad 2^3 a^{2\times 3} = 8a^6$

4. $\left(2^2 a^5 b^3\right)^2 = 2^{2\times 2} a^{5\times 2} b^{3\times 2} = 2^4 a^{10} b^6 = 16a^{10}b^6$

5. $(-3a^2)^2 = \left(-3\right)^2 \times a^{2\times 2} = 9a^4$

6. $\left(s^2 t^4\right)^3 = s^{2\times 3} t^{4\times 3} = s^6 t^{12}$

SOLUTIONS

7. $\left(\dfrac{s}{t}\right)^5 = \dfrac{s^5}{t^5}$

8. $\left(\dfrac{3^9}{3^5}\right)^2 = \left(3^4\right)^2 = 3^8$

9. $\left(\dfrac{st^8}{rt^4}\right)^2 = \left(\dfrac{st^4}{r}\right)^2 = \dfrac{s^2t^8}{r^2}$

10 $\left(\dfrac{36a^5b^8}{4a^4b^5}\right)^2 = \left(9ab^3\right)^2 = 9^2a^2b^{3\times 2} = 81a^2b^6$

#7: Negative Law of Exponents: *If the base is powered by the negative exponent, then the base becomes reciprocal with the positive exponent.*

$$x^{-m} = \frac{1}{x^{m}}$$

So, when I have a Negative Exponent, I switch the base to its reciprocal with a Positive Exponent.

Ha Ha!

If the base with the negative exponent is in the denominator, it moves to the numerator to lose its negative sign!

$$5^{-3} = \frac{1}{5^{3}} = \frac{1}{125}$$

and

$$\frac{1}{3^{-2}} = 3^{2} = 9$$

#8: Zero Law of Exponents: *Any base powered by zero exponent equals one.*

$$x^0 = 1$$

So zero factors of a base equals 1. That makes sense! Every power has a coefficient of 1.

$5^0 = 1$

and

$a^0 = 1$

and

$(5a)^0 = 1$

Try these:

1. $\left(2a^2b\right)^0 =$

2. $y^2 \times y^{-4} =$

3. $\left(a^5\right)^{-1} =$

4. $s^{-2} \times 4s^7 =$

5. $\left(3x^{-2}y^3\right)^{-4} =$

6. $\left(s^2t^4\right)^0 =$

7. $\left(\dfrac{2^2}{x}\right)^{-1} =$

8. $\left(\dfrac{3^9}{3^5}\right)^{-2} =$

9. $\left(\dfrac{s^2t^2}{s^4t^4}\right)^{-2} =$

10. $\left(\dfrac{36a^5}{4a^4b^5}\right)^{-2} =$

SOLUTIONS

1. $\left(2a^2b\right)^0 = 1$

2. $y^2 \times y^{-4} = y^{-2} = \dfrac{1}{y^2}$

3. $\left(a^5\right)^{-1} = \dfrac{1}{a^5}$

4. $s^{-2} \times 4s^7 = 4s^5$

5. $\left(3x^{-2}y^3\right)^{-4} = \left(3^{-4}x^8y^{-12}\right) = \dfrac{x^8}{81y^{12}}$

6. $\left(s^2t^4\right)^0 = 1$

SOLUTIONS

7. $\left(\dfrac{2^2}{x}\right)^{-1} = \left(\dfrac{4}{x}\right)^{-1} = \dfrac{x}{4}$

8. $\left(\dfrac{3^9}{3^5}\right)^{-2} = \left(3^4\right)^{-2} = 3^{-8} = \dfrac{1}{3^8}$

9. $\left(\dfrac{s^2t^2}{s^4t^4}\right)^{-2} = \left(s^{-2}t^{-2}\right)^{-2} = s^4t^4$

10. $\left(\dfrac{36a^5}{4a^4b^5}\right)^{-2} = 9^{-2}a^{-2}b^{10} = \dfrac{b^{10}}{81a^2}$