D.A.V. PUBLIC SCHOOL, BARIATU, RANCHI.

Class XII (2021 – 22)

Holiday Assignment in Mathematics

I) Case Study Questions:

CASE STUDY - I

A relation R on a set A is said to be an equivalence relation on A iff, R is

(i) Reflexive, i.e., a R a $\forall$ a ϵ A
(ii) Symmetric, i.e., a R b $\Rightarrow$ b R a, a $\forall$ (a, b) ϵ R
(iii) Transitive, i.e., a R b, b R c $\Rightarrow$ a R c, $\forall$ a, b, c ϵ A

Answer the following questions using this information:

1) R = { (a, b), (b, a), (c, a), (a, c)} is a relation defined on the set {a, b, c}. R is
(a)Reflexive (b) Symmetric (c) Transitive (d) None of these

2) Let A = {1, 2, 3} and R = { (1, 2), (2, 3), (1, 3) } be a relation on set A. Then R is

(a)Neither reflexive nor transitive (b) neither symmetric nor transitive

(c)Reflexive (d) Transitive

3) In the set Z of all integers, which of the following is not an equivalence relation?

(a)R : x R y $\Rightarrow$ x $\leq$ y (b) R: x R y $\Rightarrow$ x = y

(c)R: x R y $\Rightarrow$ x – y is an integer (d) All the above

4) A relation R = { (1, 1), (2, 2), (3, 3), (1, 3)} is defined on a set A = {1, 2, 3}. The ordered pair to be

Added to R so as to make it the smallest equivalence relation is

(a)(2, 1) (b) (3, 1) (c) (2, 1) (d) (2, 3)

Case Study – II

We know that a function f : A $\rightarrow$ B given by f(x) = y is bijective if f is both One – One and Onto.

Use this information to answer the following:

1) A function f is said to be One – One if

a)x = y $\Rightarrow$ f(x) = f(y) b) f(x) = f(y) $\Rightarrow$ x = y

c) x $\neq$y $\Rightarrow$ f(x) = f(y) d) f(x) $\neq$ f(y) $\Rightarrow$ x = y

2) A function $f: R - \{3\} \rightarrow R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is

a)Injective b) Surjective

c) Bijective d) injective but not Surjective

3) Modulus function is

a) One – One But not Onto

b) Onto But not One - One

c) Neither one – One nor Onto

d) Both One – one and Onto

4) Which of the following functions from Z to itself are bijections?

a) $f(x) = x^3$ b) $f(x) = x + 2$ c) $f(x) = 2x + 1$ d) $f(x) = x^2 + x$

CASE STUDY – III

If y = f(t) is a differentiable function of t and t = g(x) is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$. This is called the substitution method or Chain Rule. Use this information to solve the following questions:

1) $\frac{d}{dx}(Cos\sqrt{x})$ =

a) $\sin\sqrt{x}$ b) $-\sin\sqrt{x}$ c) $-\frac{sin\sqrt{x}}{2\sqrt{x}}$ d) $\frac{sin\sqrt{x}}{2\sqrt{x}}$

2) $\frac{d}{dx}(e^{x\,sinx})$ =

a) $e^{x\,cosx}$ b) xcos x $(e^{x\,sinx})$ c) $(e^{x\,sinx})(xcos\,x + \sin x)$ d) $(e^{x\,cosx})(xcos\,x + \sin x)$

3) $\frac{d}{dx}(\sec(logx^n)$ =

a) $\frac{secx^n tanx^n}{x^n}$ b) $\frac{n\sec(logx^n)\tan(logx^n)}{x}$ c) $\frac{n\sec(logx^n)\tan(logx^n)}{x^n}$ d) $\frac{\sec(logx^n)\tan(logx^n)}{x}$

4) $\frac{d}{dx}\left(\sqrt{x^2 + a^2}\right)$ =

a) $\frac{x}{y}$ b) $-\frac{x}{y}$ c) $\frac{y}{x}$ d) $-\frac{y}{x}$

CASE STUDY – IV

If two variables x and y are connected by the relation of the form F(x, y) = k, and it is not convenient to express y in terms of x in the form y = g(x), such a function is called an Implicit function. To find $\frac{dy}{dx}$ in case of implicit functions, we differentiate both sides with respect to x, taking the derivative of the function of y(terms of y), h(y) w.r.t. x as $\frac{d}{dy}(h(y)).\frac{dy}{dx}$.

Using this information find $\frac{dy}{dx}$ for the following functions.

1. If $x^2 + 2xy + y^3 = 41$

a) $\frac{-2(x+y)}{(2x+3y^2)}$ b) $\frac{2(x+y)}{(2x+3y^2)}$ c) $\frac{-(x+y)}{(2x+3y^2)}$ d) $\frac{(x+y)}{(2x+3y^2)}$

2. $e^{x-y} = \log\left(\frac{x}{y}\right)$

a) $\frac{y(1-xe^{x-y})}{x(ye^{x-y}-1)}$ b) $\frac{y(xe^{x-y}-1)}{x(ye^{x-y}-1)}$ c) $\frac{x(1-xe^{x-y})}{y(ye^{x-y}-1)}$ d) $-\frac{x(1-xe^{x-y})}{y(ye^{x-y}-1)}$

3. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), -1 < x < 1, x \neq 0$

a) $\frac{-x}{\sqrt{1-x^2}}$ b) $\frac{x}{\sqrt{1-x^2}}$ c) $\frac{-x}{\sqrt{1-x^4}}$ d) $\frac{x}{\sqrt{1-x^4}}$

4. $y = x\, siny$

a) $\frac{\sin y}{1-x\, cosy}$ b) $\frac{\sin y}{1-\, cosy}$ c) $\frac{\sin y}{1+x\, cosy}$ d) $\frac{x \sin y}{1-x\, cosy}$

II) SOLVE THE FOLLOWING QUESTIONS:

1) Show that $f: [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is One – One. Find the inverse of $f: [-1, 1] \to Range(f)$.
2) Let $f: N \to N\ be\ a\ function\ defined\ as\ f(x) = 9x^2 + 6x - 5$. Show that $f: N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence evaluate $f^{-1}(43)$ and $f^{-1}(163)$.
3) If $f(x) = \frac{3x-2}{2x-3}$, prove that $fof(x) = x, \forall\, x \in R - \{\frac{3}{2}\}$.
4) Prove that the composition of two bijective functions is also a bijection.
5) Show that the relation R = {(a, b): 2 divides a – b} is an equivalence relation on the set A = {0, 1, 2, 3, 4, 5}. Write the equivalence [0].
6) Write the Principal Values of the following:

   i) $\sin^{-1}(\sin(-600^0))$ ii) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ iii) $\sec^{-1}\left(2\, tan\frac{3\pi}{4}\right)$
7) Simplify : i) $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), \frac{\pi}{4} < x < \frac{5\pi}{4}$ ii) $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\}$
8) If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\frac{1}{\sqrt{3}}$, find the value of x.
9) Prove that $2\tan^{-1}\left\{tan\frac{\alpha}{2}\ tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right\} = \tan^{-1}\left(\frac{sin\alpha\ cos\beta}{\cos\alpha+\sin\beta}\right)$
10) Prove that $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$.

***