Class X

Unit I

IMPORTANT RESULTS

AND FORMULAE

**Chapter 1 :**

 **Real Numbers**

* **Euclid’s Division Algorithm / Lemma :** Let there be given two positive integers a and b(≠ 0) , a > b. Then, there exists a unique pair of integers, q and r such that a = bq + r , 0 ≤ r < b where, the integers q and r are called the quotient and remainder respectively.
* **Fundamental Theorem of Arithmetic (or Unique Prime Factorisation Theorem)** Every natural number, different from 1, is expressible as a product of primes and the expression is, apart from the order of prime factors, unique.

Or

Every composite number can be expressed as a product of primes, and this decomposition is unique, a part from the order in which the prime factors occur.

* **Theorem :** If p is a prime number and, if p divides a2, then p divides a, where a is a positive integer.
* **Decimal Expansion Theorems :** There are the following two theorems, called Decimal Expansion Theorems :

**Theorem 1**. A rational number has terminating decimal expansion if and only if the denominaton has prime factorization of the form 2m x 5n, where m, n are non zeroes positive integers.

**Or**

Let x = $\frac{p}{q}$ be a rational number, where p and q are co-prime, q≠ 0. Then, x has a termination decimal expeansion if and only if q has a prime factorization of the form 2m x 5n, where m, n are non zeroes positive integers.

**Theorem 2** if a rational number in standard form is such that the prime factorization of its denominator is not of the form 2m x 5n, where m, n are non zero positive integers, then rational number has a decimal expansion which is non terminating and recurring (repeating).

Or

Let x = $\frac{p}{q}$ be a rational number, where p and q are co-prime, q≠ 0, such that the prime factorization of q is not of the form 2m x 5n, where m, n are non zeroes positive integers. Then, x has a decimal expansion which is non terminating and recurring (repeating).

**Chapter 2 :**

**Polynomials**

* **Polynomial :** A polynomial is an expression of the form :

F(x) = a0xn + a1xn-1 + a2xn-2 + ….. + anx0

Where, ‘n’ is a non-negative integer / whole number, ‘x’ is the variable of the polynomial.

a0, a1, a2, …………. an, the constants, called the co-efficients of the polynomial f(x), are the co-efficients of the terms xn, xn-1, xn-2, ….., x0 respectively, a0 ≠ 0, co-efficient of the polynomial, an, is called the constant term.

* **Degree if a polynomial** : It is the highest exponent / power of x in f(x). In the above polynomial, deg f(x) = n.
* **Types of polynomial :** A polynomial is called a linear, quadratic, cubic or biquadratic, if its degree is 1, 2, 3 or 4 respectively.
* **Quadratic Polynomial :** A quadratic polynomial in x, with real co-efficients, is of the form :

f(x) = ax2 + bx + c, where, a,b,c, are real numbers and a≠ 0.

* **Zeroes of a Polynomial :** The zeroes of a polynomial, f(x), are precisely the x-coordinates of the points, where the graph of y= f(x) intersect the x-axis.
* **Graph of a Quadratic Polynomial :** The geometric representation of a quadratic polynomial is a curve, called parabola. The parabola opens upwards, if the co-efficient of x2 > 0. (i.e., positive). The parabola opens downwards, if the co-efficient of x2 < 0 (i.e., negative).
* **Number of Zeroes of a Polynomial :** A polynomial of degree n can have at most n zeroes. So, a quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.
* **Relationship between Zeroes and Co-efficients :** for a quadratic Polynomial,

f(x) = ax2 + bx + c, if α and β are the zeroes of f(x), then

**Sum of zeores**, S = α + β = $\frac{-b}{a}$ , and **product of zeroes**, P = αβ = $\frac{c}{a} $.

* **Division Algorithm for Polynomials :** If f(x), g(x) ≠ 0 are any two polynomials with eg. f(x) ≥ deg. G(x), then there exist, two unique polynomials q(x) and r(x) such that

f(x) = g(x). q(x) + r(x) , where either r(x) = 0, or deg. r(x) < deg. g(x).

 **Chapter 3 :**

 **Pair of Linear Equations in Two Variables**

* **A Pair of Linear Equation in Two Variables :** Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is

 a1x + b1y + c1 = 0

 a2 x + b2y + c2= 0

* Where a1, a2, b1, b2, c1, c2 are real numbers, such that a1, b1 ≠ 0 ; a22, b22 ≠ 0
* **A Pair of Linear Equations in Two Variables is called**
1. **Consistent**, if it has a unique solution and the lines on the graph intersect at a point.
2. **Inconsistent**, if it has no solution and the lines on the graph are parallel.
3. **Dependent**, if it has infinitely many (distinct common) solutions and the lines of the graph coincide.
* **Methods of Solving Linear Equations in Two Variables :** A pair of linear equations in two variables can be represented, and solved, by the

(i) graphical method (ii) algebraic method.

* **Graphical Method :** The graph of a pair of linear equations in two variables is represented by two lines :
1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is consistent.
2. If the lines coincide, then there are infinitely many solutions – each point on the line being a solution. In this case, the pair of equations is dependent (consistent).
3. If the lines are parallel, then the pair of equations has no solution. This case, the pair equations is inconsistent.
* **Algebraic Methods :** There are the following three methods for finding the solution(s) of a pair of linear equations.

(i) Substitution Mehtod (ii) Elimination Method

(iii) Cross – multiplication Method

* **Conditions of solvability of Linear Equations :** If a pair of linear equation is given by a1x + b1y + c1 = 0 and a2x + b2y + c2 = 0, then the algebraic and graphic conditions for Consistency / Solvability are, as given in the Table below :

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Sr. No | Type of Consistency | Definition | Algebraic Condition | Graphical Condition | Graphical Representation |
| 1 | **Consistent** | A unique solution | Rx ≠ Ryi.e., $\frac{a\_{1}}{a\_{2}}\ne \frac{b\_{1}}{b\_{2}}$ | Intersecting lines |  |
| 2 | **Inconsistent** | No solution | Rx = Ry ≠ Rci.e., $\frac{a\_{1}}{a\_{2}}=\frac{b\_{1}}{b\_{2}}\ne \frac{c\_{1}}{c\_{2}}$ | Parallel lines |  |
| 3 | **Dependent** | Infinitely many solutions | Rx = Ry = Rci.e., $\frac{a\_{1}}{a\_{2}}=\frac{b\_{1}}{b\_{2}}=\frac{c\_{1}}{c\_{2}}$ | Coincident lines |  |

* **Equations Reducible to a Pair of Linear Equations in Two Variables :** There an several situations which can be mathematically represented by two equations that are not linear to start with. But, we transform them so that they are reduced to a pain of linear equations.

 **Chapter 4 :**

 **Triangles**

* Similar Triangles :
1. **Definition :** Two triangles having the same shape but not necessarily the same size an called similar triangles.

Two triangles are similar, if

1. Their corresponding angles are equal.
2. Their corrensponding sides are proportional (i.e., in the same ratio).
3. All the congruent triangles are similar but the converse is not true.
4. **Criteria for Similarity of Triangles :** Various Criteria for similarity of triangles are :
5. **“AAA” Similarity Criterion :** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ration (or proportion) and hence the two triangles are similar.
6. **“AA” Similarity Criterion :** If in two triangles, two angles of one triangle are respectively equal to two angles of the other triangle, then the two triangles are similar.
7. **“SSS” Similarity Criterion :** If in two triangles, corresponding sides are proportional (i.e., in the same ratio), then their corresponding angles are equal and hence the two triangles are similar.
8. **“SAS” Similarity Criterion :** If one angle of a triangle, is equal to one angle of another triangle and the sides including these angles are proportional (i.e., in the same ratio) then the two triangles are similar.
9. **“RHS” Similarity Criterion:** If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of the other triangle, then the two triangles are similar.
* (a) **Thale’s Basic Proportionality Theorem :** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

(b) **Converse of the Basic Proportionality Theorem :** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

* **Perimeter Ratio Result of Similar Triangles :** The ratio of the perimeters of two similar triangles is equal to the ratio of their corresponding sides.
* **Area Ratio Theorem of Similar Triangles :** The ratios of the areas of two similar triangles are equal to the square of the ratio of their corresponding sides.
* **Similarity of Triangles Formed by an Altitude of Right Triangle :** If a perpendicular is drawn from the vertex of a right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
* (a) **Pythagoras Theorem** : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(b) **Converse of Pythagoras Theorem** : In a triangle if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

 **Chapter 5 :**

**Introduction to Trigonmetry and Trigonometric Identities**

* **The Trigonometric Ratios** : In fundamental right ∆OMP, t-ratios of $∠θ$ are defined as follows.
1. Sin $θ$ = $\frac{MP}{OP}=\left(\frac{Opposite side}{Hypotenuse}\right)$ P
2. Cos $θ$ = $\frac{OM}{OP}=\left(\frac{Adjacent side}{Hypotenuse}\right)$
3. tan $θ$ = $\frac{MP}{OM}=\left(\frac{Opposite side}{Adhacent side}\right)$
4. Cot $θ$ = $\frac{OM}{MP}=\left(\frac{Adjacent side}{Opposite side}\right)$
5. Sec $θ$ = $\frac{OP}{OM}=\left(\frac{Hypotenuse}{Adjacent side}\right)$ O M
6. Cosec $θ$ = $\frac{OP}{MP}=\left(\frac{Hypotenuse}{Opposite Side}\right)$
* **Resiprocal Relations :** For any acute $∠θ$,Reciprocal relations are defined as follows :

(i) Cot $θ$ = $\frac{1}{\tan(θ)}$ (ii) Sec $θ$ = $\frac{1}{\cos(θ)}$

(iii) Cosec $θ$ = $\frac{1}{\sin(θ)}$

* **Quotient Relations :** For any acute $∠θ$, Quotient relations are defined as follows :

(i) tan $θ$ = $\frac{\sin(θ)}{\cos(θ)}$ (ii) cot $θ$ = $\frac{\cos(θ)}{\sin(θ)}$

* **Constancy of t-ratios :** The t-ratios are always the same for the same angle.
* **Trigonometric Ratios of Five Specific (i.e., standard) angles :** The values of the trigonometric ratios for angles 00, 300, 450, 600 and 900 are :

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $θ$ →t-ratio | 00 | 300 | 450 | 600 | 900 |
| sin $θ$ | 0 | $$\frac{1}{2}$$ | $$\frac{1}{\sqrt{2}}$$ | $$\frac{\sqrt{3}}{2}$$ | 1 |
| cos $θ$ | 1 | $$\frac{\sqrt{3}}{2}$$ | $$\frac{1}{\sqrt{2}}$$ | $$\frac{1}{2}$$ | 0 |

Notes 1. The values of tan $θ$ can be obtained by using the quotient relation tan $θ$ = $\frac{\sin(θ)}{\cos(θ)}$ and the values of cot $θ,\sec(θ )$and cosec $θ $can be obtained by recalling the values of their reciprocal t-ratios.

 2. cosec 00, cot00, tan900 and sec900 are not defined.

* **Limits to the Values of T-ratios :**

(i) The value of sin $θ$ or cos $θ$ never exceeds 1. (i.e., sin $θ$ and cos $θ$ $≯$ 1, numerically)

(ii) The value of sec $θ$ or cosec $θ$ is always greater than or equal to 1. (i.e., sec $θ $and cosec $θ$ $\geq $ 1, numerically)

(iii) The value tan $θ$ or cot $θ$ can be any numerical value.

* **Variation in the Values of sin** $θ$ **and cos** $θ$**, 00** $\leq $$θ$$\leq $ **900 :**

(i) As $θ$ increases from 00 to 900, sin $θ$ increases from 0 to 1.

(ii) As $θ$ increases from 00 to 900, cos $θ$ decreases from 1 to 0.

* **Trigonometric Ratios of Complementary Angle :**

Complementary Formulae : The t-ratios of (900 – $θ$) are given by Com-plementary Formulae given below :

(i) sin (900 – $θ$) = cos $θ$

(ii) cos (900 – $θ$) = sin $θ$

 (iii)tan (900 – $θ$) = cot $θ$.

(iv)cot (900 – $θ$) = tan $θ$

(v)sec (900 – $θ$) = cosec $θ$

 (vi)cosec (900 – $θ$) = sec $θ$

**Note :** The above Complementary Formulae can be remembered by using the following rule.

**Aid to Memory :** For the t-ratios of the compliment of an angle.

 “Put on co, if that is absent ;

 Take off co, if that is present”.

* **Trigonometric Identities :** For all the values of $θ$ such that $θ$, 00 $\leq $ $θ$ $\leq $ 900 , the following t-identities hold :

(i) sin2$ θ$ + cos2$ θ$ = 1

(a) sin2$ θ$ = 1 – cos2$ θ$ and, (b) cos2$ θ$ = 1 – sin2$ θ$

(ii) 1 + tan2$ θ$ = sec2$ θ$

(a) tan2$ θ$ = sec2$ θ$ - 1and, (b) sec2$ θ$ - tan2$ θ$ =1

(iii) 1 + cot2$θ$ = cosce2$ θ$

 (a) cot2$ θ$ = cosec2$ θ$ – 1, (b) cosec2$ θ$ - cot2$ θ$ = 1

* **Given the Value of one t-ratio, all the other t-ratio can be determined :** If one of the t-ratios of an acute angle is known, the remaining t-ratios of the angle can be easily determined by using (i) Triangle Method (ii) Identity Method.

 **Chapter 6 :**

 **Statistics**

* **Measures of Central Tendency for Grouped Data :**

(i) **Mean :** It is computed by three methods :

(a) By Direct Method : X = $\frac{∑f\_{i}x\_{i}}{N}$

(b) By Assumed Mean Deviation Method :

 X = A + $\frac{∑f\_{i}x\_{i}}{∑f\_{i}}$

(c) By Short-cut / step – Deviation Method :

 X = A + $\frac{∑f\_{i}d\_{i}}{∑f\_{i}} x h$

**Note :** Out of the above three methods, the most conventient and time saving is the last one, i.e., Short Cut / Step - Deviation Method.

(ii) **Mode :** The mode for grouped data is computed by using the Mode Fromula :

Mode, M0 = l + $\frac{f\_{m-}f\_{i}}{2f\_{m- }f\_{i - } f\_{2}} x h$

Where, l = Lower limit of the Modal class interval

 h = Width of the Modal class interval

 fm = Frequency of the Modal Class interval

f1 = Frequency of the Pre Modal class (i.e. the class preceding the Modal class)

f2 = Frequency of the Post Modal class (i.e. the class succeeding the Modal class)

(iii) **Median :** The median for grouped data is computed by using the Median formula :

 Me = l + $\frac{h}{f\_{m}} \left(\frac{N}{2}- c\right)$

Where, l = Lower limit of the Median Class

 C = Cumulative frequency of the Median class (up to its lower limit) (i.e.) cumulative frequency of the class preceding the Median class)

 h = Width of the Median class

 fm = Simple frequency of the Median class

 N = $∑f\_{i}$ = (f1 + f2 + ….. + fn)

**Note :** The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

* **Empirical Formula :** The relationship between
* Mean (X), Median (Me) and Mode (Mo) is given by

**Karl Pearson’s Empirical Formula :**

 **Mode = 3 Median – 2 Mean**

i.e.,  **Mo = 3Me – 2X**

If any two out of mean, median and mode are known, then the third can be computed using the above Empirical Formula.

* **Ogive :** An ogive is a graphical representation of cumulative frequency distribution, in the form of a smooth, sontinuous, free-hand curve, which is either ever-rising upwards or ever falling downwards.

(i) Two types of Ogives are “<type” and “>type”.

(ii) The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives for this data.

**REMARKS** 1. For computing Mode and Median for a grouped data, ensure that the class intervals are continuous before applying the Mode, and Median formulae.

 2. For construction of an ogive, the class intervals should be continuous.

 3. In case of ogives, the scale may not be same, on both the axes.

**UNIT – II**

**IMPORTANT RESULTS AND FORMULAE**

 **Chapter 1 :**

 **Quadratic Equations**

* **Standard Form of a Quadratic Equations :** The most general form of a quadratic equation in the variable x called the standard form, is : ax2 + bx + c = 0, where a ≠ 0, a, b, c are real numbers.
* **Roots (or Solutions) of a Quadratic Equation :** Those values of x, which satisfy a quadratic equation, are called roots (or solutions) of the equation. Thus, a real number α is called a root of the quadratic equation, ax2 + bx + c = 0, if aα2 + bα + c = 0. A quadratic equation can have at most two roots, which are usually denoted by α and β.

Note : The zeores of the quadratic polynomial, ax2 + bx + c, and the roots of the quadratic equation, ax2 + bx + c = 0, are the same.

* **Discriminant of a Quadratic Equation :** It is a relationship between the coefficients of a quadratic equation and is given by Disc. D = b2 – 4ac.
* **Finding the roots of a Quadratic Equation :** There are three methods to find the roots of a quadratic equation, ax2 + bx + c = 0, a ≠ 0.
1. **By the Factorisation Method :** It is applied when the discriminant of a quadratic equation, D(b2 – 4ac) is a perfect square of a positive number. In this case, factorise ax2 +bx + c, a ≠ 0, into a product of two linear factors, and then the roots of the quadratic equation, ax2 + bx + c = 0 are found by equating each factor to zero.
2. **By the Method of Completing the Squares :** The key point involved in its 5-step procedure is the addition of [$\frac{1}{2}$ the coefficient of x]2 on both the sides, with leading coefficient unity and constant term to R.H.S.
3. **By the Quadratic Formula :** It directly gives the two roots of a Quadratic Equation, provided its discriminant D ≥ 0, by the formula stated below :

x = $\frac{- b \pm \sqrt{D}}{2a},$ where, D = b2 – 4ac

* **Nature of Roots of a Quadratic Equation :** The nature of roots of a quadratic equation, ax2 + bx + c = 0, depends upon the nature of its discriminant.
1. **If D > 0** , (i.e., positive), then the roots are **real and unequal /distinct.**
2. **If D = 0**, then the roots ; are **real and equal / coincident**.
3. **If D < 0** , then there are no real roots.

 **Chapter 2 :**

 **Arithmetic Progressions**

* **Arithmetic Progression (A.P) :** An arithmetic progression (A.P.) is an ordered list of numbers in which each term progresses (i.e., increases or decreases) successively by a constant / fixed number called, the common difference (d). Each term of an A.P, except the first term, is obtained by adding the common difference (d) to the preceding term.
* **The General Form of an A.P :** The general form of an A.P. with the first term, a, and common difference, d is given by **: a, a + d, a + 2d, a + 3d, …..**
* **The General (or nth) Term of an A.P :** In an A.P with first term a and common difference, d, the General term (or nth term) is given by Tn = a + (n – 1) d.
* **Selection of Terms in an A.P :**
* (i) **for three terms**, take : a – d, a and a +d
* (ii) **for four terms**, take : a – 3d, a – d, a + d and a + 3d (iii) **For five terms**, take : a – 2d, a – d, a, a + d and a + 2d and so on.
* **The Sum Formula :** The sum of the first, n terms of an A.P is given by

Sn = $\frac{n}{2} $[ 2a + (n – 1) d] ………… 1st Form

Sn = $\frac{n}{2} $(a + 1) ………… 2nd Form

Where, l is the last term in Sn

i.e., l= Tn = a + (n – 1) d

**REMARK :** In the above Sum Formula, use the 1st Form when the common difference, d is known and use the 2nd Form when the last term, l is known.

* **nth Term is terms of Sn :** If the sum, Sn of n terms of an A.P, is given, then the nth term of the A.P can be computed by using the formula Tn = Sn – Sn-1, for n > 1.

 **Chapter 3 :**

 **Circles**

* **Concept of the Tangent (s) to a circle**

**Def :** A tangent to a circle is the limiting position of a secant when its two points intersection with the circle coincide.

**The point where the tangent touches the circle is called point of contact.**

**Def :** A secant to a circle is a line which intersects it in two distinct points.

 Thus, the basic difference between a secant and a tangent is that whilst secant to a circle intersects it in two distinct points, a tangent to a circle intersects it in exactly one point.

* **“Tangent Radius” Theorem :** The tangent at any point of a circle is perpendicular to the radius through the point of contact.
* **“Equal Tangent Lengths” Theorem :** The length of the two tangents from an external point to a circle are equal.

 **Chapter 4 :**

 **Constructions**

* **Division of a line segment** in a given ratio and its justification by using Thale’s – Basic Proportionality Theorem.
* **Construction of a triangle** similar to given triange, as per the given scale factor which may be less than 1 or greater than 1 and its justification by using the concept of similar triangles.
1. If the scale factor is less than 1, then the required triangle is smaller than the given triangle.
2. If the scale factor is more then 1, then the required triangle is larger than the given triangle.
* **Construction of a pair or tangents** from an external point to a circle and its justification by using the concepts of :
1. An angle in a semi-circle is right angle
2. “Tangent Radius” Theorem of a circle, i.e., Tangent ┴ Radius.

 **Chapter 5 :**

**Some Applications of Trigonometry**

(Heights and Distances)

* **Trigonometry** finds extensive applications in many branches of science. It is profitably applied in solving problems of determining heights and distances. In this conncetion, the following definitions are required.
1. The line of Vision (or the Line of Sight) : It is the line drawn from the eye of an observer to a point in the object where the observer is viewing the object.
2. The Horizontal Line : It is the line drawn parallel to the Earth’s surface throught he eye of an observer looking straight.
3. Angle of Elevation of an Object : It is the angle formed by the line of vision with the horizontal when the object is above the horizontal level (i.e., the case when we elevate i.e., raise our head to look at the object.)
4. Angle of Depression of an Object : It is the angle formed by the line of vision with the horizontal when the object is below the horizontal level (i.e., the case when we depress i.e., lower our head to look at the object.)
5. The Height of an Object : The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.
* **Rule to find one side of a right-angled triangle when another side and an acute angle are given (the hypotenuse also being regarded as a side).**

 $\frac{Required side}{Given side }$ = A certain t-ratio of the given angle.

 **Chapter 6 :**

 **Probability**

* **The Difference Between Experimental Probability and Theoretical Probability :** Whilst, **experimental** (or **empirical**) probability of an event is based on the results of actual experiments (i.e. what has actually happened), the theoretical (or classical) probability of the event attempts to make predictions on the basis of certain assumptions.
* **Probability : Def.** The theoretical (or classical) probability of an event E, written as P(E), is defined as

P(E) = $\frac{Number of outcomes favourable to E}{Number of all possible outcomes of the experiment}$

It is assumed that the outcomes of the experiment are equally likely.

* **Sure Event :** The probability of a sure event (or certain event) is 1.
* **Impossible Event :** The probability of an impossible event is 0.
* **Range if Probability :** The probability of an event E is a number P(E) such that 0 ≤ P(E) ≤ 1
* **Elementary Event :** An event having only one outcome is called an elementary event.
* **Sum of Probabilities :** The sum of the probabilities of all the elementary events of an experiment is 1.

For any event E, P(E1) + P(E) = 1 $⟹$ P(E1) = 1 – P (E)

Where, E1 stands for ‘not E’. E and E1are called complementary events.

 **Chapter 7 :**

 **Co-ordinate Geometry**

 **(Lines in Two Dimensions)**

* **Distance Formula :** The distance between the points P(x1, y1) and Q(x2, y2) is

PQ = $\sqrt{(x\_{2}- x\_{1})^{2}+(y\_{2}- y\_{1})^{2}}$

* **Distance of a point from the Origin :** The distance of a point P(x1, y1) from the origin, O, is
	+ - * OP = $\sqrt{x\_{1}^{2}+ y\_{1}^{2})^{2}}$

**Section Formula :** The co-ordinates of the point P(x, y) which divides the join of A(x1, y1) and B(x2, y2) internally in the ration m1 : m2, are

$$x=\frac{m\_{1}x\_{2}+ m\_{2}x\_{1}}{m\_{1}+ m\_{2}} , y= \frac{m\_{1}y\_{2}+ m\_{2}y\_{1}}{m\_{1}+ m\_{2}}$$

**Mid-point Formula :** The co-ordinates of the mid-point

 P(x, y) of the join of A(x1, y1) and B(x2, y2) is

$$x=\frac{x\_{1}+ x\_{2}}{2} , y= \frac{y\_{1}+ y\_{2}}{2}$$

* **Area Formula for A Triangle :** The area of the triangle formed by the points A(x1,y1) and B(x2,y2) and C(x3, y3) is

ar(∆ABC = $\frac{1}{2}$ [x1 (y2 - y3) + x2(y3 – y1) + x3(y1 – y2)]…… 1st form

 = $\frac{1}{2}$ [(x1y2-x2y1) + (x2y3 – x3y2) + (x3y1 – x1y3)] … 2nd form

**Condition for Collinearity of Three Points :** If the area of the triangle ABC is zero, then three points A, B and C are collinear (i.e., they lie on a line).

i.e., x1(y2-y3) + x2(y3-y1) + x3(y1-y2) =0 ……. 1st form

or (x1y2 – x2y1) +(x2y3 – x3y2) + (x3y1 – x1y3) = 0 ……. 2nd form

* **Area Formula for Quadrilateral :** The area of the quadrilateral formed by the points A(x1,y1), B(x2,y2), C(x3,y3) and D(x4, y4), taken in order, is

Ar(∆ABCD) = = $\frac{1}{2}$ [(x1y2-x2y1) + x2y3 – x3y2) + (x3y4 – x4y3) + (x4y1 – x1y4)]

 **Chapter 8 :**

 **Areas Related to Circles**

* **For a circle of radius r, we have**

(**i) Perimeter / Circumference of a circle, C = 2πr**

**(ii) Area of a circle, A = πr2**

* **For a semi circle :**

**(i) Perimeter of the semi circle, = (πr + 2r)**

**(ii) Area of the semi circle, =** $\frac{1}{2} $**πr2**

* **Quadrant of a circle :**

 **(i) Perimeter of a quadrant of a circle, =**$\frac{1}{4}$ **(2πr ) + 2r**

$\left(\frac{πr}{2}+ 2r\right)$

 **(ii) Area of a quadrant of a circle, =** $\frac{1}{4} $**(πr)2**

 **Area of a ring** = π(R + r) (R – r) unit2

* **Circular – wheel in Motion :**

**Number of revolution in 1 minute**,

N = $\frac{Distance travelled by the wheel in 1 miniute}{Perimeter/ Circumference of the wheel}$

i.e., **n =** $\frac{D}{2πr}$

* **For a Sector of a Circle :**

 **(i) Length of an arc of a sector, l =** $\frac{2πr}{360} X θ$

 **(ii) Area of a sector, A =** $\frac{πr^{2}}{360}X θ$ **…. 1st Form**

 **=** $\frac{1}{2} X l X r$ **…. 2nd Form**

 **(iii) Perimeter of a Sector, P = 2r +** $\left(\frac{2πrθ}{360}\right)$

* **Area of Segments :**
* **Area of a Segment = (Area of the corresponding sector) – ( Area of the corresponding ∆)**
1. **Area of a minor-segment = r2** $\left[\frac{πθ}{360}-\frac{1}{2} sinθ\right]$ **when θ is an acute ∠) ….1st form**

**= r2** $\left[\frac{πθ}{360}-sin\frac{θ}{2} cos\frac{θ}{2}\right]$ **when θ is an obtuse ∠) ….2nd form**

 **(b) Area of a major –segment = (Area of the circle) – ( Area of the minor segment)**

Note : In the above, letters l, r, θ, n, R, A, C, D etc., have been used in their usual meaning, depending on the context.

 **Chapter 9 : Surface Areas and Volumes**

* **Combination of Solids :** When any two of the basic solids, namely, cuboid, cone, cylinder and sphere are combined, a new solid is forme.
* **Surface Areas of Combination of Solids :** The total surface areas of combinations of solids are obtained by adding together the exposed portions of the surface areas of the individual solids, and not by adding together their total surface areas.
* **Volumes of Combinations of Solids :**The volumes of combinations of solids are actually the sum of the volumes of the constituents forming the new solids.
* **Conversion of solids :** When one solid is converted into another, the volume of material in the original solid equals the volume of material in the second.
* **Frustum of a Right Circular Cone :** When a right circular cone is cut (or sliced through) by a plane parallel to its base, through some points on its axis and the smaller conical portion containing the vertex is removed, then the resulting left out portion of the solid, (which is in the form of a bucket) is called a Frustum of a Right Circular Cone. It consists of two unequal flat circular bases and a curved surface.
* **Formulae for the Frustum of a cone :** The formulae involving the frustum of a cone are :
1. **Volume of the Frustum of a Cone,**

**V =** $\frac{1}{3}$ **πh(R2 + r2 + Rr)**

1. **Curved Surface Area of the Frustum of a Cone,**

**S = πl (R + r)**

 **Where, l =** $\sqrt{h^{2}+( R-r)^{2} } $

1. **Total Surface Area of the frustum of a cone,**

**St = [πR2 + πr2 + π(R + r)]**

 **= πl (R + r) + πR2 + πr2**

**Where, l =** $\sqrt{h^{2}+( R-r)^{2} }$

**(iv) Total Surface Area of the frustum of a cone which is open at one of the bases,**

 **St = πl (R + r) + πr2**

**Or St = πl ( R + r) + π (R2 + 7πr2),**

**[Depending on the end which is open]**

REMARKS 1. The capacity of a bucket is equal to its volume.

 2. The above formulae for the frustum of a cone can also be used for a bucket with R > r